

GCE

Mathematics

Advanced GCE

Unit 4726: Further Pure Mathematics 2

Mark Scheme for January 2011

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$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$				
	1	$t = \tan \frac{1}{2}x \Rightarrow dt = \frac{1}{2}\sec^2 \frac{1}{2}x dx = \frac{1}{2}(1+t^2) dx$	B1	For correct result AEF (may be implied)
		$\int \frac{1}{1 + \frac{1}{x^2}} dx = \int \frac{1}{1 + \frac{1}{x^2}} \frac{2}{x^2} dt$	M1	For substituting throughout for <i>x</i>
		$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	A1	For correct unsimplified <i>t</i> integral
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		$= \int \frac{1}{1+t} \mathrm{d}t = \ln\left 1+t\right (+c)$	M1	
2 (i) $f(x) = \tanh^{-1}x, f'(x) = \frac{1}{1-x^2}, f''(x) = \frac{2x}{(1-x^2)^2}$		$= \ln k \left 1 + \tan \frac{1}{2} x \right (+c)$	A1 5	
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$			5	windy of numerical, o is not required
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	2 (i)	$f(x) = \tanh^{-1} x$, $f'(x) = \frac{1}{1 - x^2}$, $f''(x) = \frac{2x}{(1 - x^2)^2}$	M1	For quoting $f'(x) = \frac{1}{1 \pm x^2}$ and attempting to
$ \begin{array}{c} f'''(x) = \\ \frac{2(1-x^2)^2 - 2x \cdot 2(1-x^2) \cdot -2x}{(1-x^2)^4} OR \frac{2x \cdot 4x}{(1-x^2)^3} + \frac{2}{(1-x^2)^2} & \text{M1} \\ \frac{2(1-x^2)^2 + 8x^2(1-x^2)}{(1-x^2)^4} OR \frac{8x^2}{(1-x^2)^3} + \frac{2(1-x^2)}{(1-x^2)^3} \\ = \frac{2(1+3x^2)}{(1-x^2)^3} & \text{A1} & \text{For correct unsimplified } f'''(x) \\ \frac{2(1+3x^2)}{(1-x^2)^3} & \text{A1} & \text{For simplified } f'''(x) \text{ www AG} \\ \\ (ii) & f(0) = 0, f'(0) = 1, f''(0) = 0 & \text{B1} \sqrt{\frac{1}{N}} & \text{For all values correct (may be implied)} \\ f'''(0) = 2 \Rightarrow \tanh^{-1}x = x + \frac{1}{3}x^3 & \text{For correct series} \\ \hline (b) & \frac{1}{y} = \frac{5ax}{x^2 + a^2} \Rightarrow yx^2 - 5ax + a^2y = 0 & \text{M1} & \text{For expressing as a quadratic in } x \\ \hline MI & \text{For expressing as a quadratic in } x \\ \hline MI & \text{For correct range} \\ \hline METHOD 1 & \text{For correct range} \\ \hline METHOD 2 & \text{M2} + \frac{5ax}{x^2 + a^2} \Rightarrow \frac{dy}{dx} = \frac{-5a(x^2 - a^2)}{(x^2 + a^2)^2} & \text{M1} & \text{For correct range} \\ \hline METHOD 2 & \text{M2} + \frac{5ax}{x^2 + a^2} \Rightarrow \frac{dy}{dx} = \frac{-5a(x^2 - a^2)}{(x^2 + a^2)^2} & \text{M1} & \text{For correct range} \\ \hline METHOD 2 & \text{M2} + \frac{5ax}{x^2 + a^2} \Rightarrow \frac{dy}{dx} = \frac{-5a(x^2 - a^2)}{(x^2 + a^2)^2} & \text{M1} & \text{For correct values of } x \\ \hline M1 & \text{For correct range} \\ \hline M2 & \text{M2} + \frac{5ax}{x^2 + a^2} \Rightarrow \frac{dy}{dx} = \frac{-5a(x^2 - a^2)}{(x^2 + a^2)^2} & \text{M3} + \frac{1}{x^2 + a^2} & \text{For correct values of } x \\ \hline M3 & \text{For correct range} \\ \hline M4 & \text{For correct range} \\ \hline M5 & \text{M3} + \frac{1}{x^2 + a^2} & \text{For correct range} \\ \hline M6 & \text{M4} + \frac{1}{x^2 + a^2} & \text{M4} & \text{For correct range} \\ \hline M6 & \text{M5} + \frac{1}{x^2 + a^2} & \text{M6} + \frac{1}{x^2 + a^2} & \text{M7} \\ \hline M1 & \text{For correct range} \\ \hline M2 & \text{For correct maximum } f.t. from (i)(b) \\ \hline M3 & \text{M3} + \frac{1}{x^2 + a^2} & \text{M4} & \text{For correct maximum } f.t. from (i)(b) \\ \hline M4 & \text{M4} + \text{For correct minimum } f.t. from (i)(b) \\ \hline M2 & \text{M4} + \text{For correct minimum } f.t. from (i)(b) \\ \hline M3 & \text{M4} + \text{For correct minimum } f.t. from (i)(b) \\ \hline M4 & \text{M4} + \text{For correct minimum } f.t. from (i)(b) \\ \hline M4 & \text{M4} + \text{M4} + \text{M4} \\ \hline M4 & \text{M5} + \text{M4} + \text{M4} \\ \hline M4 & \text{M5} + \text{M4} + \text{M4} \\ \hline M4 & \text{M5} + \text{M4} + \text$				• /
		f'''(x) =	A1	For $f''(x)$ correct WWW
		` '	M1	For using quotient <i>OR</i> product rule on $f''(x)$
$=\frac{2(1+3x^2)}{(1-x^2)^3} \qquad \qquad \text{A1 5 For simplified } f''(x) \text{ www AG}$ $(ii) f(0) = 0, \ f'(0) = 1, \ f''(0) = 0 \qquad \qquad \text{B1} \checkmark \qquad \text{For all values correct (may be implied)} \\ f.t. \ from \ (i) \qquad \qquad \qquad \text{For evaluating } f'''(0) \ \text{ and using Maclaurin} \\ expansion \qquad \qquad \qquad \qquad \text{For correct series}$ $\boxed{8}$ $3 \ (i)(a) \text{Asymptote } y = 0 \qquad \qquad \text{B1 1 For correct equation (allow x-axis)}$ $\boxed{b} \frac{\text{METHOD 1}}{y = \frac{5ax}{x^2 + a^2}} \Rightarrow yx^2 - 5ax + a^2y = 0 \qquad \qquad \text{M1 For using } b^2 - 4ac \leq 0$ $\boxed{b^2 \geqslant 4ac \Rightarrow 25a^2 \geqslant 4a^2y^2 \Rightarrow -\frac{5}{2} \leqslant y \leqslant \frac{5}{2}} \qquad \text{A1 For } 25a^2 - 4a^2y^2 \text{ seen or implied}$ $\boxed{\text{METHOD 2}} \qquad \qquad$		$\frac{1-x^2}{(1-x^2)^4} OK \frac{1-x^2}{(1-x^2)^3} + \frac{1}{(1-x^2)^4}$	$\overline{)^2}$ A1	For correct unsimplified $f'''(x)$
(ii) $f(0) = 0, f'(0) = 1, f''(0) = 0$ $f'''(0) = 2 \Rightarrow \tanh^{-1} x = x + \frac{1}{3}x^{3}$ (b) $\frac{1}{y} = \frac{5ax}{x^{2} + a^{2}} \Rightarrow yx^{2} - 5ax + a^{2}y = 0$ $\frac{b^{2}}{x^{2} + a^{2}} \Rightarrow \frac{1}{2} \Rightarrow$		$= \frac{2(1-x^2)^2 + 8x^2(1-x^2)}{(1-x^2)^4} OR \frac{8x^2}{(1-x^2)^3} + \frac{2(1-x^2)}{(1-x^2)^3}$		
(ii) $f''(0) = 0, f'(0) = 1, f'(0) = 0$ $f'''(0) = 2 \Rightarrow \tanh^{-1} x = x + \frac{1}{3}x^{3}$ $\frac{M1}{8} \text{For evaluating } f'''(0) \text{ and using Maclaurin expansion}$ A1 3 For correct series $\frac{3 \text{ (i)(a)}}{8} \text{Asymptote } y = 0$ B1 1 For correct equation (allow x-axis) $\frac{b}{y} = \frac{5ax}{x^{2} + a^{2}} \Rightarrow yx^{2} - 5ax + a^{2}y = 0$ $\frac{b^{2} \geqslant 4ac \Rightarrow 25a^{2} \geqslant 4a^{2}y^{2} \Rightarrow -\frac{5}{2} \leqslant y \leqslant \frac{5}{2}}{(x^{2} + a^{2})^{2}}$ METHOD 2 $y = \frac{5ax}{x^{2} + a^{2}} \Rightarrow \frac{dy}{dx} = \frac{-5a(x^{2} - a^{2})}{(x^{2} + a^{2})^{2}}$ M1* For correct range $\frac{dy}{dx} = 0 \Rightarrow x = \pm a \Rightarrow y = \pm \frac{5}{2}$ A1 For correct values of x For differentiating y by quotient OR product rule $\frac{dy}{dx} = 0 \Rightarrow x = \pm a \Rightarrow y = \pm \frac{5}{2}$ A1 For correct values of x For finding y values and giving argument for range For correct range $\frac{dy}{dx} = 0 \Rightarrow x = \pm a \Rightarrow y = \pm \frac{5}{2}$ A1 For correct range $\frac{dy}{dx} = 0 \Rightarrow x = \pm a \Rightarrow y = \pm \frac{5}{2}$ A1 For correct range $\frac{dy}{dx} = 0 \Rightarrow x = \pm a \Rightarrow y = \pm \frac{5}{2}$ A1 For correct values of x For finding y values and giving argument for range For correct range $\frac{dy}{dx} = 0 \Rightarrow x = \pm a \Rightarrow y = \pm \frac{5}{2}$ A1 For correct equation (allow x-axis) $\frac{dy}{dx} = 0 \Rightarrow x = \pm a \Rightarrow y = \pm \frac{5}{2}$ A1 For correct equation (allow x-axis) $\frac{dy}{dx} = 0 \Rightarrow x = \pm a \Rightarrow y = \pm \frac{5}{2}$ A1 For correct equation (allow x-axis) $\frac{dy}{dx} = 0 \Rightarrow x = \pm a \Rightarrow y = \pm \frac{5}{2}$ A1 For correct equation (allow x-axis) $\frac{dy}{dx} = 0 \Rightarrow x = \pm a \Rightarrow y = \pm \frac{5}{2}$ A1 For correct equation (allow x-axis) $\frac{dy}{dx} = 0 \Rightarrow x = \pm a \Rightarrow y = \pm \frac{5}{2}$ A1 For correct equation (allow x-axis) $\frac{dy}{dx} = 0 \Rightarrow x = \pm a \Rightarrow y = \pm \frac{5}{2}$ A1 For correct equation (allow x-axis) $\frac{dy}{dx} = 0 \Rightarrow x = \pm a \Rightarrow y = \pm \frac{5}{2}$ A1 For correct equation (allow x-axis) $\frac{dy}{dx} = 0 \Rightarrow x = \pm a \Rightarrow y = \pm \frac{5}{2}$ A2 For correct equation (allow x-axis)		$=\frac{2(1+3x^2)}{(1-x^2)^3}$	A1 5	For simplified $f'''(x)$ www AG
$f'''(0) = 2 \Rightarrow \tanh^{-1} x = x + \frac{1}{3}x^3$ $\begin{array}{c} M1 & \text{For evaluating } f'''(0) \text{ and using Maclaurin expansion} \\ A1 & 3 & \text{For correct series} \\ \hline 8 \\ \hline \\ \textbf{8} \\ \hline \\ \textbf{3 (i)(a)} & \text{Asymptote } y = 0 \\ \hline \textbf{(b)} & \frac{\text{METHOD I}}{y = \frac{5ax}{x^2 + a^2}} \Rightarrow yx^2 - 5ax + a^2y = 0 \\ \hline & b^2 \geqslant 4ac \Rightarrow 25a^2 \geqslant 4a^2y^2 \Rightarrow -\frac{5}{2} \leqslant y \leqslant \frac{5}{2} \\ \hline & A1 & \text{For expressing as a quadratic in } x \\ \hline & \textbf{M1} & \text{For using } b^2 - 4ac \leqslant 0 \\ \hline & b^2 \geqslant 4ac \Rightarrow 25a^2 \geqslant 4a^2y^2 \Rightarrow -\frac{5}{2} \leqslant y \leqslant \frac{5}{2} \\ \hline & A1 & \text{For correct range} \\ \hline & \textbf{METHOD 2} \\ \hline & y = \frac{5ax}{x^2 + a^2} \Rightarrow \frac{dy}{dx} = \frac{-5a\left(x^2 - a^2\right)}{\left(x^2 + a^2\right)^2} \\ \hline & A1 & \text{For differentiating } y \text{ by quotient } \textit{OR product rule} \\ \hline & \frac{dy}{dx} = 0 \Rightarrow x = \pm a \Rightarrow y = \pm \frac{5}{2} \\ \hline & A1 & \text{For correct values of } x \\ \hline & A2 & \text{For correct values of } x \\ \hline & A3 & \text{For correct values of } x \\ \hline & A4 & \text{For correct range} \\ \hline & \text{(ii)(a)} & y = 0 \\ \hline & B1 & 1 & \text{For correct equation (allow } x \text{-axis}) \\ \hline & \text{(ii)} & \text{(ii)} & y = 0 \\ \hline & B1 & 1 & \text{For correct equation (allow } x \text{-axis}) \\ \hline & \text{(ii)} & \text{(a)} & y = 0 \\ \hline & B1 & 1 & \text{For correct equation (allow } x \text{-axis}) \\ \hline & \text{(b)} & \text{Maximum } \sqrt{\frac{5}{2}}, \text{ minimum } -\sqrt{\frac{5}{2}} \\ \hline & B1 \sqrt{2} & \text{For correct maximum f.t. from (i)(b)} \\ \hline & \text{Allow decimals} \\ \hline & \text{(c)} & x \geqslant 0 \\ \hline & B1 & 1 & \text{For correct set of values (allow in words)} \\ \hline \end{array}$	(ii)	f(0) = 0, f'(0) = 1, f''(0) = 0	В1√	
3 (i)(a) Asymptote $y = 0$ B1 1 For correct series (b) METHOD 1 $y = \frac{5ax}{x^2 + a^2} \Rightarrow yx^2 - 5ax + a^2y = 0$ M1 For expressing as a quadratic in x M1 For using $b^2 - 4ac \leq 0$ $b^2 \geqslant 4ac \Rightarrow 25a^2 \geqslant 4a^2y^2 \Rightarrow -\frac{5}{2} \leqslant y \leqslant \frac{5}{2}$ A1 For $25a^2 - 4a^2y^2$ seen or implied METHOD 2 $y = \frac{5ax}{x^2 + a^2} \Rightarrow \frac{dy}{dx} = \frac{-5a(x^2 - a^2)}{(x^2 + a^2)^2}$ M1* For differentiating y by quotient OR product rule $\frac{dy}{dx} = 0 \Rightarrow x = \pm a \Rightarrow y = \pm \frac{5}{2}$ A1 For correct values of x For finding y values and giving argument for range Asymptote, sketch etc $\Rightarrow -\frac{5}{2} \leqslant y \leqslant \frac{5}{2}$ A1 For correct values of x For correct equation (allow x -axis) B1 I For correct equation (allow x -axis) For correct maximum f.t. from (i)(b) Allow decimals (c) $x \geqslant 0$ B1 I For correct set of values (allow in words)		$f'''(0) = 2 \rightarrow \tanh^{-1} x = x + 1 x^3$	M1	
3 (i)(a)Asymptote $y = 0$ B11For correct equation (allow x-axis)(b)METHOD 1 $y = \frac{5ax}{x^2 + a^2} \Rightarrow yx^2 - 5ax + a^2y = 0$ M1 M1For expressing as a quadratic in x For using $b^2 - 4ac \leq 0$ $b^2 \geqslant 4ac \Rightarrow 25a^2 \geqslant 4a^2y^2 \Rightarrow -\frac{5}{2} \leqslant y \leqslant \frac{5}{2}$ A1 A1For $25a^2 - 4a^2y^2$ seen or implied A1METHOD 2M1*For differentiating y by quotient OR product rule $\frac{dy}{dx} = 0 \Rightarrow x = \pm a \Rightarrow y = \pm \frac{5}{2}$ M1*For correct values of x M1For finding y values and giving argument for range(ii)(a) $y = 0$ B1 A1 For correct maximum f.t. from (i)(b) Allow decimals(b)Maximum $\sqrt{\frac{5}{2}}$, minimum $-\sqrt{\frac{5}{2}}$ B1 $\sqrt{\frac{5}{2}}$ For correct maximum f.t. from (i)(b) Allow decimals(c) $x \geqslant 0$ B11For correct set of values (allow in words)		$1 (0) - 2 \rightarrow \tanh x - x + \frac{1}{3}x$	A1 3	
(b) METHOD 1 $y = \frac{5ax}{x^2 + a^2} \Rightarrow yx^2 - 5ax + a^2y = 0$ $y = \frac{5ax}{x^2 + a^2} \Rightarrow yx^2 - 5ax + a^2y = 0$ M1 For expressing as a quadratic in x M1 For using $b^2 - 4ac \leq 0$ $\frac{b^2 \geqslant 4ac \Rightarrow 25a^2 \geqslant 4a^2y^2 \Rightarrow -\frac{5}{2} \leqslant y \leqslant \frac{5}{2}}{A1} = \frac{A1}{A1} = A1$			8	
(ii) $y = \frac{5ax}{x^2 + a^2} \Rightarrow yx^2 - 5ax + a^2y = 0$ $y = \frac{5ax}{x^2 + a^2} \Rightarrow yx^2 - 5ax + a^2y = 0$ $y = \frac{5ax}{x^2 + a^2} \Rightarrow 25a^2 \geqslant 4a^2y^2 \Rightarrow -\frac{5}{2} \leqslant y \leqslant \frac{5}{2}$ $y = \frac{5ax}{x^2 + a^2} \Rightarrow \frac{dy}{dx} = \frac{-5a(x^2 - a^2)}{(x^2 + a^2)^2}$ $\frac{dy}{dx} = 0 \Rightarrow x = \pm a \Rightarrow y = \pm \frac{5}{2}$ $A1 \qquad \text{For correct range}$ $A1 \qquad \text{For differentiating } y \text{ by quotient } OR \text{ product rule}$ $\frac{dy}{dx} = 0 \Rightarrow x = \pm a \Rightarrow y = \pm \frac{5}{2}$ $A1 \qquad \text{For correct values of } x$ $A1 \qquad \text{For finding } y \text{ values and giving argument for range}$ $A1 \qquad \text{For correct equation (allow } x - axis)$ $A1 \qquad \text{For correct maximum } f.t. \text{ from (i)(b)}$ $A1 \qquad \text{For correct maximum } f.t. \text{ from (i)(b)}$ $A1 \qquad \text{For correct maximum } f.t. \text{ from (i)(b)}$ $A1 \qquad \text{For correct minimum } f.t. \text{ from (i)(b)}$ $A1 \qquad \text{For correct set of values (allow in words)}$	3 (i)(a)		B1 1	For correct equation (allow <i>x</i> -axis)
$y = \frac{1}{x^2 + a^2} \Rightarrow yx^2 - 5ax + a^2y = 0$ $b^2 \geqslant 4ac \Rightarrow 25a^2 \geqslant 4a^2y^2 \Rightarrow -\frac{5}{2} \leqslant y \leqslant \frac{5}{2}$ $METHOD 2$ $y = \frac{5ax}{x^2 + a^2} \Rightarrow \frac{dy}{dx} = \frac{-5a(x^2 - a^2)}{(x^2 + a^2)^2}$ $\frac{dy}{dx} = 0 \Rightarrow x = \pm a \Rightarrow y = \pm \frac{5}{2}$ $A1 \text{For correct range}$ $\frac{dy}{dx} = 0 \Rightarrow x = \pm a \Rightarrow y = \pm \frac{5}{2}$ $\text{A1} \text{For differentiating } y \text{ by quotient } OR \text{ product rule}$ $\frac{dy}{dx} = 0 \Rightarrow x = \pm a \Rightarrow y = \pm \frac{5}{2}$ $\text{A1} \text{For correct values of } x$ $\text{M1} \text{For finding } y \text{ values and giving argument for range}$ $\text{A1} \text{For correct range}$ $\text{A1} \text{For correct range}$ $\text{A2} \text{For correct range}$ $\text{A3} \text{For correct range}$ $\text{A4} \text{For correct range}$ $\text{A5} \text{For correct range}$ $\text{A6} \text{For correct range}$ $\text{A1} \text{For correct range}$ $\text{A2} \text{For correct range}$ $\text{A3} \text{For correct range}$ $\text{A4} \text{For correct range}$ $\text{A5} \text{For correct range}$ $\text{A6} \text{For correct range}$ $\text{A1} \text{For correct range}$ $\text{A2} \text{For correct maximum f.t. from (i)(b)}$ $\text{A3} \text{A4} \text{A4} \text{For correct maximum f.t. from (i)(b)}$ $\text{A1} \text{A2} \text{A3} \text{A3} \text{A4} \text{A4} \text{A4} \text{A4} \text{A4} \text{A5}$ $\text{A5} \text{A6} \text{A7} \text{A7} \text{A7} \text{A7} \text{A8} \text{A8} \text{A9} \text$	(b)		M1	For expressing as a quadratic in x
METHOD 2 $y = \frac{5ax}{x^2 + a^2} \Rightarrow \frac{dy}{dx} = \frac{-5a(x^2 - a^2)}{(x^2 + a^2)^2}$ M1* For differentiating y by quotient OR product rule $\frac{dy}{dx} = 0 \Rightarrow x = \pm a \Rightarrow y = \pm \frac{5}{2}$ A1 For correct values of x M1 For finding y values and giving argument for range A1 For correct range (*dep) (ii)(a) $y = 0$ B1 1 For correct equation (allow x-axis) (b) Maximum $\sqrt{\frac{5}{2}}$, minimum $-\sqrt{\frac{5}{2}}$ B1 For correct maximum f.t. from (i)(b) B1 Por correct maximum f.t. from (i)(b) Allow decimals (c) $x \geqslant 0$ B1 1 For correct set of values (allow in words)		$y = \frac{1}{x^2 + a^2} \implies yx^2 - 5ax + a^2y = 0$	M1	For using $b^2 - 4ac \leq 0$
METHOD 2 $y = \frac{5ax}{x^2 + a^2} \Rightarrow \frac{dy}{dx} = \frac{-5a(x^2 - a^2)}{(x^2 + a^2)^2}$ M1* For differentiating y by quotient OR product rule $\frac{dy}{dx} = 0 \Rightarrow x = \pm a \Rightarrow y = \pm \frac{5}{2}$ Al For correct values of x M1 For finding y values and giving argument for range Asymptote, sketch etc $\Rightarrow -\frac{5}{2} \leqslant y \leqslant \frac{5}{2}$ Al For correct range (*dep) (ii)(a) $y = 0$ Bl 1 For correct equation (allow x-axis) (b) Maximum $\sqrt{\frac{5}{2}}$, minimum $-\sqrt{\frac{5}{2}}$ Bl $\sqrt{\frac{1}{2}}$ For correct maximum f.t. from (i)(b) Allow decimals (c) $x \geqslant 0$ Bl 1 For correct set of values (allow in words)		$b^2 > 4ac \rightarrow 25a^2 > 4a^2v^2 \rightarrow 5 < v < 5$	A1	For $25a^2 - 4a^2y^2$ seen or implied
$y = \frac{5ax}{x^2 + a^2} \Rightarrow \frac{dy}{dx} = \frac{-5a\left(x^2 - a^2\right)}{\left(x^2 + a^2\right)^2}$ $\frac{dy}{dx} = 0 \Rightarrow x = \pm a \Rightarrow y = \pm \frac{5}{2}$ Al For correct values of x M1 For finding y values and giving argument for range Asymptote, sketch etc $\Rightarrow -\frac{5}{2} \leqslant y \leqslant \frac{5}{2}$ Al For correct range $(*dep)$ (ii)(a) $y = 0$ Bl 1 For correct equation (allow x -axis) (b) Maximum $\sqrt{\frac{5}{2}}$, minimum $-\sqrt{\frac{5}{2}}$ Bl $\sqrt{\frac{5}{2}}$ Bl $\sqrt{\frac{5}{2}}$ For correct maximum f.t. from (i)(b) Allow decimals (c) $x \geqslant 0$ Bl 1 For correct set of values (allow in words)			A1 4	For correct range
$\frac{-3y}{dx} = 0 \Rightarrow x = \pm a \Rightarrow y = \pm \frac{3}{2}$ $\text{Asymptote, sketch etc} \Rightarrow -\frac{5}{2} \leqslant y \leqslant \frac{5}{2}$ $\text{M1} \qquad \text{For finding } y \text{ values and giving argument for range}$ For correct range $(*dep)$ $\text{(ii)(a)} y = 0$ $\text{B1} \textbf{1} \text{For correct equation (allow } x\text{-axis})$ $\text{(b)} \text{Maximum} \sqrt{\frac{5}{2}}, \text{ minimum} -\sqrt{\frac{5}{2}}$ $\text{B1} \sqrt{\textbf{2}} \text{For correct maximum f.t. from (i)(b)}$ $\text{B1} \sqrt{\textbf{2}} \text{For correct minimum f.t. from (i)(b)}$ Allow decimals $\text{(c)} x \geqslant 0$ $\text{B1} \textbf{1} \text{For correct set of values (allow in words)}$		(2 2)	M1*	• • • •
Asymptote, sketch etc $\Rightarrow -\frac{5}{2} \leqslant y \leqslant \frac{5}{2}$ (ii)(a) $y = 0$ B1 1 For correct equation (allow x-axis) (b) Maximum $\sqrt{\frac{5}{2}}$, minimum $-\sqrt{\frac{5}{2}}$ B1 $\sqrt{2}$ For correct maximum f.t. from (i)(b) Allow decimals (c) $x \geqslant 0$ B1 1 For correct set of values (allow in words)		$\frac{dy}{dy} = 0 \Rightarrow x = +a \Rightarrow y = +\frac{5}{2}$		
Asymptote, sketch etc $\Rightarrow -\frac{3}{2} \leqslant y \leqslant \frac{3}{2}$ (ii)(a) $y = 0$ B1 1 For correct equation (allow x-axis) (b) Maximum $\sqrt{\frac{5}{2}}$, minimum $-\sqrt{\frac{5}{2}}$ B1 $\sqrt{\frac{5}{2}}$ For correct maximum f.t. from (i)(b) B1 $\sqrt{\frac{5}{2}}$ For correct minimum f.t. from (i)(b) Allow decimals (c) $x \geqslant 0$ B1 1 For correct set of values (allow in words)		ux	MI	
(ii)(a) $y = 0$ B1 1 For correct equation (allow x-axis) (b) Maximum $\sqrt{\frac{5}{2}}$, minimum $-\sqrt{\frac{5}{2}}$ B1 $\sqrt{\frac{5}{2}}$ For correct maximum f.t. from (i)(b) B1 $\sqrt{\frac{5}{2}}$ For correct minimum f.t. from (i)(b) Allow decimals (c) $x \ge 0$ B1 1 For correct set of values (allow in words)		Asymptote, sketch etc $\Rightarrow -\frac{1}{2} \leqslant y \leqslant \frac{1}{2}$		
Maximum $\sqrt{\frac{2}{2}}$, minimum $-\sqrt{\frac{2}{2}}$ B1 $\sqrt{2}$ For correct minimum f.t. from (i)(b) Allow decimals (c) $x \ge 0$ B1 1 For correct set of values (allow in words)	(ii)(a)	y = 0		For correct equation (allow <i>x</i> -axis)
(c) $x \ge 0$ B1 1 For correct set of values (allow in words)	(b)	Maximum $\sqrt{\frac{5}{2}}$, minimum $-\sqrt{\frac{5}{2}}$		For correct minimum f.t. from (i)(b)
9	(c)	$x \geqslant 0$	B1 1	
			9	

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4 (i)	$8\sinh^4 x = \frac{8}{16} \left(e^x - e^{-x} \right)^4$	B1	$\sinh x = \frac{1}{2} \left(e^x - e^{-x} \right) \text{ seen or implied}$
	$\equiv \frac{8}{16} \left(e^{4x} - 4e^{2x} + 6 - 4e^{-2x} + e^{-4x} \right)$	M1	For attempt to expand $\left(\ldots\right)^4$
	$\equiv \frac{1}{2} \left(e^{4x} + e^{-4x} \right) - \frac{4}{2} \left(e^{2x} + e^{-2x} \right) + \frac{6}{2}$	M1	by binomial theorem OR otherwise For grouping terms for $\cosh 4x$ or $\cosh 2x$
	$\equiv \cosh 4x - 4\cosh 2x + 3$	A1 4	OR using e^{4x} or e^{2x} expressions from RHS For correct expression AG
-	SR may be done wholly from RHS to LHS	M1 M1	Evidence of factorising required for 2nd M1
		B1 A1	C 1
(ii)	METHOD 1 $\cosh 4x - 3\cosh 2x + 1 = 0$	3.54	
	$\Rightarrow (8\sinh^4 x + 4\cosh 2x - 3) - 3\cosh 2x + 1 = 0$	M1	For using (i) and $\cosh 2x = \pm 1 \pm 2 \sinh^2 x$
	$\Rightarrow 8\sinh^4 x + 2\sinh^2 x - 1 = 0$	A1 M1	For correct equation
	$\Rightarrow (4\sinh^2 x - 1)(2\sinh^2 x + 1) = 0 \Rightarrow \sinh x = \pm \frac{1}{2}$	A1	For solving their quartic for sinh <i>x</i> For correct sinh <i>x</i> (ignore other roots)
	$\Rightarrow x = \ln\left(\pm\frac{1}{2} + \frac{1}{2}\sqrt{5}\right) = \pm\ln\left(\frac{1}{2} + \frac{1}{2}\sqrt{5}\right)$	$A1\sqrt{5}$	For correct answers (and no more) f.t. from their value(s) for sinh x
	SR Similar scheme for $8\cosh^4 x - 1$	$4\cosh^2 x$	$+5 = 0 \Rightarrow \cosh x = \frac{1}{2}\sqrt{5} \Rightarrow x = \pm \ln\left(\frac{1}{2} + \frac{1}{2}\sqrt{5}\right)$
	METHOD 2 $\cosh 4x - 3\cosh 2x + 1 = 0$		
	$\Rightarrow (2\cosh^2 2x - 1) - 3\cosh 2x + 1 = 0$	M1	For using $\cosh 4x = \pm 2 \cosh^2 2x \pm 1$
	$\Rightarrow 2\cosh^2 2x - 3\cosh 2x = 0$	A1	For correct equation
	$\Rightarrow \cosh 2x = \frac{3}{2} \Rightarrow x = \frac{1}{2} \ln \left(\frac{3}{2} \pm \frac{1}{2} \sqrt{5} \right)$	M1	For solving for $\cosh 2x$
	$= \pm \frac{1}{2} \ln \left(\frac{3}{2} + \frac{1}{2} \sqrt{5} \right)$	A1 A1√	For correct $\cosh 2x$ (ignore others)
	$-\pm\frac{1}{2}\operatorname{Im}\left(\frac{1}{2}+\frac{1}{2}\operatorname{V}_{3}\right)$	111 (For correct answers (and no more) f.t. from value(s) for cosh 2x
	METHOD 3 Put all into exponentials	M1	For changing to $e^{\pm kx}$
	$\Rightarrow e^{4x} - 3e^{2x} + 2 - 3e^{-2x} + e^{-4x} = 0$	A1	For correct equation
	\Rightarrow $(e^{4x} + 1)(e^{4x} - 3e^{2x} + 1) = 0$	M1	For solving for e^{2x}
	()()	A1	For correct e^{2x} (ignore others)
	\Rightarrow $e^{2x} = \frac{1}{2}(3 \pm \sqrt{5}) \Rightarrow x = \frac{1}{2}\ln\left(\frac{3}{2} \pm \frac{1}{2}\sqrt{5}\right)$	A1	For correct answers (and no more)
			f.t. from value(s) for e^{2x}
		9	
	$x_n^3 - 5x_n + 3 \qquad 2x_n^3 - 3$	M1	For attempt at N-R formula
5 (i)	$x_{n+1} = x_n - \frac{x_n^3 - 5x_n + 3}{3x_n^2 - 5} = \frac{2x_n^3 - 3}{3x_n^2 - 5}$	A1 A1 3	For correct N-R expression For correct answer (necessary details
	n n	711 3	needed) AG
	——————————————————————————————————————	3.61	Allow omission of suffixes
(ii)	$F'(x) = \frac{1}{2}(x^2 + x^2) + \frac{1}{2}(x^2 + x^2)$	M1	For using quotient OR product rule to find $F'(x)$
	$\frac{6x^{2}(3x^{2}-5)-6x(2x^{3}-3)}{6x^{2}(2x^{3}-3)} = \frac{6x(x^{3}-5x+3)}{6x^{2}(2x^{3}-5)}$	M1	For factorising numerator to show
	$\frac{6x^2(3x^2-5)-6x(2x^3-3)}{(3x^2-5)^2} = \frac{6x(x^3-5x+3)}{(3x^2-5)^2}$		$k\left(x^3-5x+3\right)$
			\
	$F'(\alpha) = \frac{6\alpha(\alpha^3 - 5\alpha + 3)}{(3\alpha^2 - 5)^2} = 0 \text{ since } \alpha^3 - 5\alpha + 3 = 0$	A1 3	For correct explanation of AG
(iii)	$x_1 = 2 \Rightarrow 1.85714, 1.83479, 1.83424, 1.83424$	B1	First iterate correct to at least 4 d.p. $OR \frac{13}{7}$
	$(\alpha =) 1.8342$	B1	For 2 equal iterates to at least 4 d.p.
	CD For starting value leading to an other	B1 3	For correct α to 4 d.p.
	SR For starting value leading to another root allow up to B1 B1 B0		Allow answer rounding to 1.8342 SR If not N-R, B0 B0 B0
	•	9	SK II liot IV-K, DO DO DO
		<u>-</u>	

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A1

6 (1)	$y = x^x \Rightarrow \ln y = x \ln x \Rightarrow \frac{1}{y} \frac{dy}{dx} = 1 + \ln x$
	$\frac{\mathrm{d}y}{\mathrm{d}x} = x^x (1 + \ln x) = 0 \implies \ln x = -1 \implies x = \mathrm{e}^{-1}$

M1For differentiating $\ln y OR x \ln x$ w.r.t. x

For $(1 + \ln x)$ seen or implied For correct x-value from fully correct **A**1 working AG

 $A > 0.2 \times 0.5^{0.5} + 0.2 \times 0.7^{0.7} + 0.1 \times 0.9^{0.9}$ (ii)

For areas of 3 lower rectangles M1

 $\Rightarrow A > 0.3881(858) > 0.388$

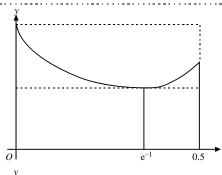
For lower bound rounding to AG **A**1

(iii) $A < 0.2 \times 0.7^{0.7} + 0.2 \times 0.9^{0.9} + 0.1 \times 1^{1}$ M1 For areas of 3 upper rectangles

 $\Rightarrow A < 0.4377(177) < 0.438$

A1 For upper bound rounding to 0.438

(iv)



M1Consider rectangle of height $f(e^{-1})$

Α1 Use at least 1 lower rectangle, height $f(e^{-1})$

B1 3 Use at least 1 upper rectangle, height f(0)

> **SR** If more than one rectangle is used for either bound, they must be shown correctly

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7 (i) $\cos 3\theta = \cos(-3\theta)$ OR $\cos \theta = \cos(-\theta)$ for all θ

M1 For a correct procedure for symmetry related to the equation OR to $\cos 3\theta$

⇒ equation is unchanged, so symmetrical about

For correct explanation relating to equation **A**1

 $r = 0 \Rightarrow \cos 3\theta = -1$

M1 For obtaining equation for tangents A1 for any 2 values A1

(ii) $\Rightarrow \theta = \pm \frac{1}{3}\pi, \pi$

A1 A1 for all, no extras (ignore outside range)

(iii)

$$\int_{-\frac{1}{3}\pi}^{\frac{1}{3}\pi} \frac{1}{2} (1 + \cos 3\theta)^2 (d\theta)$$

$$= \frac{1}{2} \int_{-\frac{1}{3}\pi}^{\frac{1}{3}\pi} 1 + 2\cos 3\theta + \cos^2 3\theta d\theta$$

В1 For correct integral with limits soi (limits may be $\left| 0, \frac{1}{3}\pi \right|$ at any stage)

 $= \frac{1}{2} \int_{-\frac{1}{2}\pi}^{\frac{1}{3}\pi} 1 + 2\cos 3\theta + \frac{1}{2} (1 + \cos 6\theta) d\theta$

M1* For multiplying out, giving at least 2 terms

For integration to M1

$$= \frac{1}{2} \left[\theta + \frac{2}{3} \sin 3\theta + \left(\frac{1}{2} \theta + \frac{1}{12} \sin 6\theta \right) \right]_{1}^{\frac{1}{3} \pi}$$

 $A\theta + B\sin 3\theta + C\sin 6\theta$ **AEF** For completing integration and substituting M1

 $= \frac{1}{2} \left[\theta + \frac{2}{3} \sin 3\theta + \left(\frac{1}{2} \theta + \frac{1}{12} \sin 6\theta \right) \right]_{-\frac{1}{2}\pi}^{\frac{1}{3}\pi}$

their limits into terms in $\frac{\cos n\theta}{\sin n\theta}$ (*dep)

 $=\frac{1}{2}\pi$

A1 5 For correct area www

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8	(i)	METHOD 1	M 1	
	` '	$\sinh\left(\cosh^{-1}2\right) =$	M1	For appropriate use of $\sinh^2 \theta = \cosh^2 \theta - 1$
		$\sinh \beta = \sqrt{\cosh^2 \beta - 1} = \sqrt{2^2 - 1} = \sqrt{3}$	A1 2	For correct verification to AG
		METHOD 2	M1	For attempted use of \ln forms of $\sinh^{-1} x$
		$\sinh^{-1}\sqrt{3} = \ln(\sqrt{3} + 2), \cosh^{-1}2 = \ln(2 + \sqrt{3})$		and $\cosh^{-1} x$
		$\Rightarrow \sinh(\cosh^{-1} 2) = \sqrt{3}$	A1	For both ln expressions seen
		METHOD 3		
		$\cosh^{-1} 2 = \ln\left(2 + \sqrt{3}\right)$	M1	For use of ln form of $\cosh^{-1} x$ and
		$\sinh\left(\cosh^{-1}2\right) = \frac{1}{2} \left(e^{\ln\left(2+\sqrt{3}\right)} - e^{-\ln\left(2+\sqrt{3}\right)}\right)$	A1	definition of $\sinh x$ For correct verification to AG
		$\sin(\cos(x)) = 2 \left(\frac{1}{2} \left(\frac{1}{2} \right) + \frac{1}{2} \left($		SR Other similar methods may be used
		$= \frac{1}{2} \left(2 + \sqrt{3} - \left(2 - \sqrt{3} \right) \right) = \sqrt{3}$		Note that $\ln(2+\sqrt{3}) = -\ln(2-\sqrt{3})$
	(ii)	$I_n = \int_0^\beta \cosh^n x \mathrm{d}x$	M1*	For attempt to integrate $\cosh x \cdot \cosh^{n-1} x$
		$= \left[\sinh x \cdot \cosh^{n-1} x\right]_0^{\beta} - \int_0^{\beta} \sinh^2 x \cdot (n-1) \cosh^{n-2} x dx$		by parts For correct first stage of integration (ignore limits)
		$= \sinh \beta \cdot \cosh^{n-1} \beta - (n-1) \int_0^\beta \left(\cosh^2 x - 1 \right) \cosh^{n-2} x$	$a dx \frac{M1}{(*dep)}$	For using $\sinh^2 x = \cosh^2 x - 1$
		$=2^{n-1}\sqrt{3}-(n-1)(I_n-I_{n-2})$	A1	For $(n-1)(I_n - I_{n-2})$ correct
		$= 2 \sqrt{3 - (n-1)(I_n - I_{n-2})}$	B1	For $2^{n-1}\sqrt{3}$ correct at any stage
		$\Rightarrow n I_n = 2^{n-1} \sqrt{3} + (n-1)I_{n-2}$	A1 6	For correct result AG
	(iii)	$I_1 = \int_0^\beta \cosh x dx = \sinh \beta = \sqrt{3}$	B1	For correct value
		$I_3 = \frac{1}{3} \left(2^2 \sqrt{3} + 2\sqrt{3} \right) = 2\sqrt{3}$	M1	For using (ii) with $n = 3 OR n = 5$
		13 - 3(2 + 3 + 2 + 3) - 2 + 3	A1	For $I_3 = \frac{1}{3} \left(2^2 \sqrt{3} + 2I_1 \right)$
				$OR \ I_5 = \frac{1}{5} \left(2^4 \sqrt{3} + 4I_3 \right)$
		$I_5 = \frac{1}{5} \left(2^4 \sqrt{3} + 8\sqrt{3} \right) = \frac{24}{5} \sqrt{3}$	A1 4	For correct value
			12	

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